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PARAMETER SETTING OF A DYNAMIC EQUATION FOR A PRODUCTION PROCESS WITH PHASE TRANSITION

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Abstract. *In the previous studies, we have been analyzing the production process evaluation by utilizing the potential function. We proposed appropriate parameters for a dynamic equation to constrain the phase transition reported in a previous study. The parameters were based on a long experience in production. The rate of return was calculated from the estimates of production orders from September 2014 to September 2016. The parameters of the dynamic equation were empirically obtained from the rate of return data. We confirmed the validity of the parameter setting by applying it to a real production process. There is no research specifying the parameters of the dynamic equation defined by a free energy of Ginzburg-Landau (GL) based on real data. We also report the change in entropy with regard to volatility. Finally, we reported the entropy of three states: a stable state, a state with an assumed phase transition, and a state with a phase transition.*
rate of return, potential function, Ginzburg−Landau (GL) free energy, entropy, phase−field method

1. **Introduction.** Conventionally, the phase transition phenomenon is widely known as a physical phenomenon. First of all, the simulated annealing method is considered to be an efficient optimization technique in the field of statistical mechanics[1]. Christopher G. Langton is known for his 1990 study of artificial beings[2]. He also conceptualized the idea of the " Edge of Chaos. " In physical phenomena, the " Edge of Chaos " refers to a phenomenon that corresponds to the transition state that exists between fluid and solid phases. Phenomena similar to the " Edge of Chaos " occur during the period from the entry of the manufacturing order for a product to its delivery. In our previous paper, when an order for manufacturing is received, there exists an outflow of cash due to the purchase of materials for the person receiving the order, and there is a lead time until cash is injected at the end of the manufacturing period[3]. We also indicated that the phase transition phenomenon is observed in the process throughput of the manufacture of certain control equipment. We verify the phase transition in the system through experiments on a production flow system[3]. Moreover, we have reported that the propagation of fluctuations corresponds to a fluctuation in the lead time by applying Burgers ' equation of fluid dynamics, which constrains the state variables in an internal process[4].

With respect to the presence of fluctuations f^{-1}, we revealed the presence of fluctuations f^{-1} by applying the spectral analysis of the rate of return deviation from this dynamic model equation. In addition, under the condition of the power spectrum of this fluctuation, it had a Lorentz spectrum. To recognize the condition, that is, we reported to lead improved manufacturing throughput by carrying out the bottleneck synchronization[5].

To increase the rate of return, it is important to reduce the lead time from a financial perspective. In addition, the rate of return is decreased if opportunities are lost and if there are excessive inventory stocks. From a practical perspective, it is necessary to synchronize the speeds of individual manufacturing operations[6]. We consider that the synchronization of manufacturing processes will lead to the improvement of the production throughput. Here, the synchronization of manufacturing processes is one method to enable the efficient progress of each process in order to increase the throughput.

On the other hand, unlike the study by T. Tanabe and M. lshikawa[7], we attempted to analyze the phase transition mechanism in the manufacturing industry by treating manufacturing processes as a closed process when seen as a single manufacturing process, that is, a process on which external forces do not act. We instead defined order parameters within a manufacturing process and further introduce the Ginzburg−Landau (GL) free energy[3, 8].

The rate of return considerably varies in response to stochastic external forces. For example, considerable delays may arise in the production process or in areas such as logistics. When analyzed by the GL potential energy, the rate of return is influenced by logistical delays, lead times, and breakdowns in electronic component. Herein, we analyzed the parameters of the potential function using the GL free energy. The rate of return was calculated from the estimates of production orders from September 2014 to September 2016. The parameters of the dynamic equation were empirically obtained from the rate of return data. By specifying these parameters, the potential function and the entropy of the three states could be obtained. The state of a real production process could be specified.

2. **Production systems in the manufacturing equipment industry.** The production methods used in manufacturing equipment are briefly covered in this paper. More information is provided in our report. More information is provided in our report[3]. This system is considered to be a " Make-to-order system with version control, " which enables manufacturing after orders are received from clients, resulting in " volatility " according to its delivery date and lead time. In addition, there is volatility in the lead time, depending on the content of the make-to-order products (production equipment).

In Fig.1(A), the " Customer side " refers to an ordering company and " Supplier (D) " means the target company in this paper. The product manufacturer, which is the source of the ordered manufacturing equipment presents an order that takes into account the market price. In Fig.1(B), the market development department at the customer 's factory receives the order through the sale contract based on the predetermined strategy.

Figure 2 illustrates a company 's decision-making process. The business monitors perceived demand trends. When a customer order is received, the perceived trend is analyzed. Based on the analysis, the company is able to decide how to respond to the analyzed demand.

3. **Production flow process.** Figure 3 depicts a manufacturing process that is termed as a production flow process. This manufacturing process is employed in the production of control equipment. In this example, the production flow process consists of six stages. In each step S1−S6 of the manufacturing process, materials are being produced.

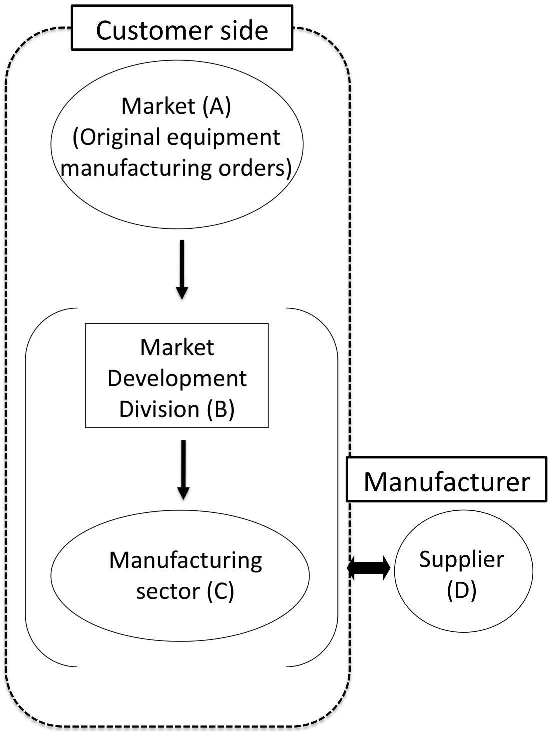

Figure 1. Business structure of company of research target

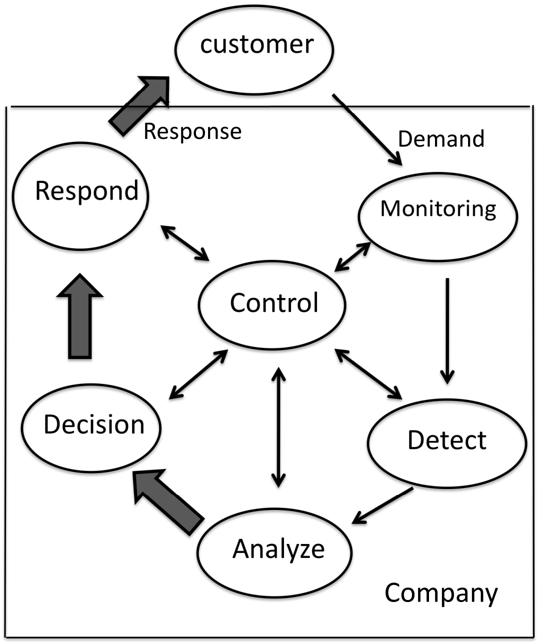

Figure 2. Decision-making process within the company

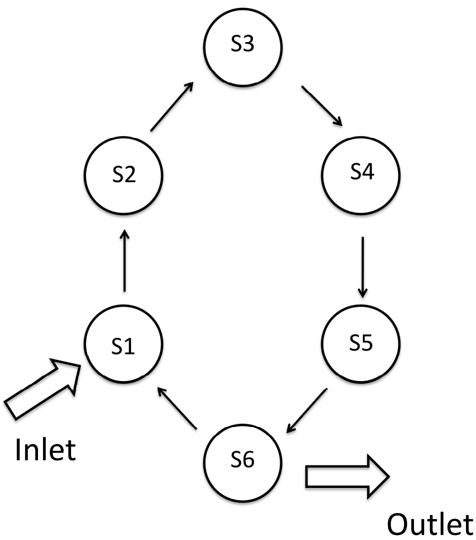

Figure 3. Production flow process

The direction of the arrows represents the direction of the production flow. In this process, production materials are supplied through the inlet and the end-product is shipped from the outlet.

For this flow production system, we make the following two assumptions:

Assumption 3.1. *The production structure is nonlinear.*

Assumption 3.2. *The production structure is a closed structure; that is, the production is driven by a cyclic system (production flow system).*

Assumption 3.1 indicates that the determination of the production structure is considered a major factor, which includes the generation value of production or the throughput generation structure in a stochastic manufacturing process (hereafter called the manufacturing field). Because such a structure is at least dependent on the demand, it is considered to have a nonlinear structure.

Because the value of such a product depends on the throughput, its production structure is nonlinear. Therefore, Assumption 3.1 reflects the realistic production structure and is somewhat valid. Assumption 3.2 is completed in each step and flows from the next step until stage S6 is completed. Assumption 3.2 is reasonable because new production starts from S1. Please refer Appendix A.

4. **Potential energy and rate of return of production process.** The description that deviation of free energy produces a return will be made.

Assumption 4.1. *Return is created by liquidity of production density function $S_i(t)$. From this, there exists a potential that depends on a production density function.*

Here, the size of potential $F(S_i(t))$ is attributed to inclination of a production density function related to a production unit, that is, liquidity. Therefore, the following equation is

Definition 4.1.

$$\frac{dF(S_i(t))}{dS_i} = -\kappa \times grand \;\; S_i(t). \tag{1}$$

where, κ is a constant.

The structure of potential in production density function $S_i(t)$ will be examined[9, 10]. Potential in the present research is defined as " ability to create a return ".

By such definition, meaning of equation (1) has been made clear. In other words, it is considered that inclination related to a production unit of potential of production field $\{S_i(t)\}$ reduces in proportion to inclination related to a production unit of production density function $S_i(t)$, resulting in creating a return (it is considered as a difference between potentials). When considering like this, we define potential energy (free energy) in a production field as follows:

Definition 4.2. *Potential energy in production field*

$$\begin{aligned}&[Potential\ of\ production\ field\ per\ production\ density]\\&= [Potential\ for\ production\ unit]\\&+ [Fluctuation\ of\ potential\ for\ production\ unit]\end{aligned}$$

Such Definition is almost equivalent to definition of the potential or free energy of a field in physics.

A transition to the dynamic state, which can be modeled by the Hamilton-Jacobi equation, requires excitation energy, which increases the free energy of the system[11].

To retain profitability in business, a continual input of products to the static field must be present. At the same time, order information must be supplied in the same manner. Figure 4 gives an overview of this production field concept[15]. The number of production units at each stage of a production unit i shifts over time. To function effectively, a production process requires a minimum number of personnel. This situation constitutes a shortest path problem. Production units can be considered to be physically located in mechanical fixtures. The production dynamics enable a company to profit from its business. We consider that revenues are generated by the displacement of the potential in the production field. In other words, the entropy increase contributed by the production unit is another source of revenue. This is the principle of maximum entropy[16].

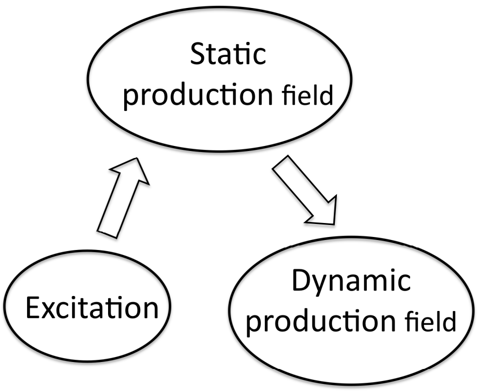

FIGURE 4. Overview of the production field concept

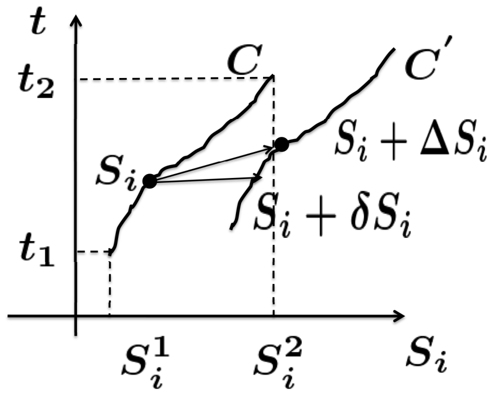

FIGURE 5. Transition from a lower-energy production process to the next process

Fig.5 illustrates the transition from a lower−energy production process (energy state C) to the (higher−energy) next process (energy state C'). In Figure 5, the number of production units at each stage of a production unit i shifts over time. To function effectively, a production process requires a minimum number of personnel. This situation constitutes a shortest path problem. The displacement of the potential in the production field generates a revenue. From the principle of maximum entropy, the entropy increase contributed by the production unit is another source of revenue[15]. We now derive the model equation that constrains the dynamic behavior of the production cost. If the production field sets $\{S_i(t)\}, i = 1, \cdots, n$, introducing sustainable order information and exciting the system with a sustainable target allows the process to progress from a static to a dynamic production field. The free energy of the process is increased by this transition[11].Please refer more detail information in our previous paper[15].

Definition 4.3. *Production cost $S_i(t)$*

$$S_i(t) \equiv S_i^*(t) \pm \Delta S_i(t) \tag{2}$$

where the production cost $S_i^*(t)$ incorporates cost fluctuations.

Definition 4.4. *The rate of return specifies the variation of the production cost; That is, the rate of return $h(t)$ generated by improvement expenditure is as follows:*

$$h(t) \cong \frac{dS_i}{dt} \tag{3}$$

5. **Parameter setting of dynamic equations constraining the rate of return function.** Let $h(t)$ be the rate of return in the presence of a phase transition phenomenon. $h(t)$ can be expressed as follows:

$$h(t) = \begin{cases} 1 & (A) \\ 0 & (B) \\ -1 & (C) \end{cases} \tag{4}$$

where A represents the positive rate of return, B represents the zero rate of return, and C represents the negative rate of return. $-1 < h(t) < 1$

A zero rate of return represents a disorderly state. Further, we obtained the following equation based on the GL free energy[11]:

Here we describe the GL free energy in a manufacturing industry as follows:

Definition 5.1. *Free energy: $F(h)$ related to production quantity*

$$F(h) = \int_0^L [\frac{r}{2}(\nabla h)^2 + W(h)]dx \tag{5}$$

Equation (5) indicates that free energy given by the space integration of a function depends on order parameter h and is GL free energy. ∇h represents fluctuations.

From here on, $h(t, x)$ is the order parameter (rate of return) which depends almost time only. It is important for the rate of return that a high quality product is completed until planned period. Therefore, we consider the rate of return to $h(t)$.

At the observation time $t \in [0, T]$, the probability function $P(t)$ has the following probability density function in the range $x \leq n(t) \leq x + dx$ as follows.

Definition 5.2. *Probability function $P(t)$*

$$P(t) = \int_{-\infty}^t \phi(x)dx \tag{6}$$

6. **Entropy analysis of rate of return $h(t)$.** We describe the state equation before discussing entropy.

Definition 6.1. *Production density $C(t, x)$*

$$\frac{\partial C(t,x)}{\partial t} = \mathcal{L}_x C(t,x) \tag{7}$$

where, t and x denote a time and stage number of process. The initial condition and boundary condition are as follows:

$$C(0, x) = C_0(x) \tag{8}$$

$$C(t, x)|_{x \in \partial\Omega} = 0 \tag{9}$$

where $\partial\Omega$ denotes a start and end process.

Then, we define a stochastic variable for the process time series variable.

Definition 6.2. *Stochastic variable $n(t)$ for the process time series variable*

$$\frac{dn(t)}{dt} = -\nu n(t) + F_R(t) \tag{10}$$

where, ν and $F_R(t)$ denote an average and an exogenous and endogenous disturbance, which are logistics delay, changing delivery date of customer and staff manufacturing mistake, etc.

Here, Probability of $n(t)$ will enter $n \sim n + dn$ is as follows: To satisfy the probability that $n(t)$ falls into $n \to n + dn$, it is to satisfy the following Fokker Planck equation:

$$\frac{\partial P(n,t)}{\partial t} = -\nu \frac{\partial (n,t)}{\partial n} + \frac{\partial^2 P(n,t)}{\partial n^2} \tag{11}$$

There is no problem even if the Langevin type equation is simplified to a normal probability type differential equation. Because, Langevin type equation can be regarded as a diffusion system[13, 14].

Assumption 6.1. *Stochastic differential equation of Normal type $n(t)$*

$$dn(t) = \mu_\xi dt + \sigma_\xi dZ_\xi(t) \tag{12}$$

where, μ_ξ, σ_ξ and $Z_\xi(t)$ are the average, volatility and Wiener process respectively.

Assumption 6.2. *$\phi(t)$ denotes a probability density function of normal type the with average zero.*

$$\phi(t) \equiv \frac{1}{\sqrt{2\pi}\sigma_\xi} \exp(-\frac{t^2}{2\sigma_\xi^2}) \tag{13}$$

where, σ_ξ denotes a volatility.

Then, we define the entropy as follows:

Definition 6.3. *Entropy S*

$$S = -\int P(t) \ln P(t) dt \tag{14}$$

As $n(t)$ is the stochastic function, we define the variable U as follows:

Definition 6.4. *Stochastic function U*

$$U =< n(t) > +\xi = n + \xi \tag{15}$$

where, $< n(t) >$ and ξ denote the average n and white noise respectively[17, 18]. The probability of existence $P(U > \theta)$ relative to the threshold θ is as follows:

$$P(U > \theta) = P(\xi > \theta - n) = P(\xi > \sigma_\xi) \tag{16}$$

Therefore,

$$\begin{aligned} P(\xi > \sigma_\xi) &= \frac{1}{\sqrt{2\pi}\sigma_\xi} \int_\xi^\infty \exp(-\frac{s^2}{2\sigma_\xi^2}) ds = \frac{1}{\sqrt{2\pi}} \int_{\xi/\sigma_\xi}^\infty \exp(-\frac{\alpha^2}{2}) d\alpha \\ &= 1 - \Phi(\xi/\sigma_\xi) \end{aligned} \tag{17}$$

where, $\sigma_\xi = \theta - n$.

Therefore, we obtain from Equations (16) and (17) as follows:

$$P(U > \theta) = 1 - \Phi(\xi/\sigma_\xi) \tag{18}$$

Here, let $\delta = \xi/\sigma_\xi$. Then, we obtain as follows:

$$\Phi(\delta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\delta \exp(-\frac{s^2}{2}) ds \tag{19}$$

where, σ_ξ denotes the volatility of ξ.

Then, we define the entropy function for threshold[11, 16].

Definition 6.5. *Entropy function for threshold S_δ*

$$S_\delta = -\int P(\delta)\ln P(\delta)d\delta \tag{20}$$

where, $P(\delta)$ denotes the probability for the threshold.

From Equation (20), we present the numerical calculations to Section 7.

7. Numerical simulation.

7.1. **Potential function using the phase field method.** Figures 6 and 7 show the rates of return of the control panel based on cost accounting in cases wherein $h_{max} \cong 0.5$ and $h_{max} \cong 0.32$, respectively. Figure 8 shows the period before improvement, and the areas marked by circles are those in which the rate of return is particularly poor. Figure 9 shows the improvement in the rate of return.

Figures 10 to 12 show the potential function graphs at parameter settings of a, b, and c. The cost calculations on which the rate of return was based used the data of the orders received between September 2014 and December 2014. These are shown in Figures 6 and 7 and Figures 8 and 9.

Figures 13 to 15 show the graphs obtained from the parameters a, b, and c, which were empirically obtained from Figures 6 and 7 and Figures 8 and 9. Figures 13 and 14 show that the period before the process improvement can be assumed to contain a potential phase transition. In contrast, Figure 15 shows the period after the process improvement, which is characterized by a stable potential.

Figure 16 shows a process transition diagram derived by applying Equations (23) and (24). Based on Equation (20), Figures 17 to 19 show the entropy values. $S_i > S_f > S_{Local}$ represents the rate of return.

Table 1. Profit margin before/after improvement of processes

	Before improvement(S_{Local})	Current process (S_f)	After improvement(S_i)
μ	2.04	6.6	2.02
σ	3.7	5.3	1.79
rate of return	0.15 $\sim$ 0.3	-0.1 $\sim$ -0.3	0.2 $\sim$ 0.3

In Table 1, $\{S_f\}$, $\{S_{Local}\}$, and $\{S_i\}$ denote the processes before improvement, during improvement $\{S_f\}$, and after improvement, respectively. $\{S_{Local}\} \subset \{S_f\}$ and $\{S_f\} \rightarrow \{S_i\}$.

7.2. **Probabilistic representation of process time series.** From Figure 16, we define the deviation of entropy as follows:

Definition 7.1.

$$\Delta S(t_1, t_2) = S(t_1) - S(t_2) \tag{21}$$

Then, S_i and S_f are derived as follows:

$$S_i = S[P_i] + S[\{P_i, P_0\}] \tag{22}$$

$$S_f = S[P_f] + S[\{P_i, P_0\}] \tag{23}$$

$$\begin{aligned} S_f - S_i &= \Big[S[P_f] - S[P_i]\Big] + \Big[S[\{P_i, P_0\}] - S[\{P_f, P_0\}]\Big] \\ &= S[P_f] - S[P_i] \end{aligned} \tag{24}$$

where, $\Big[S[\{P_i, P_0\}] - S[\{P_f, P_0\}]\Big] = 0.$

As a results, we obtain as follows:

$$S_f - S_i \cong 6.298 \tag{25}$$

$$S_f - S_{Local} = -4.7572 \tag{26}$$

where, $\Delta S[P_i, P_f|P_0] \to 0.$

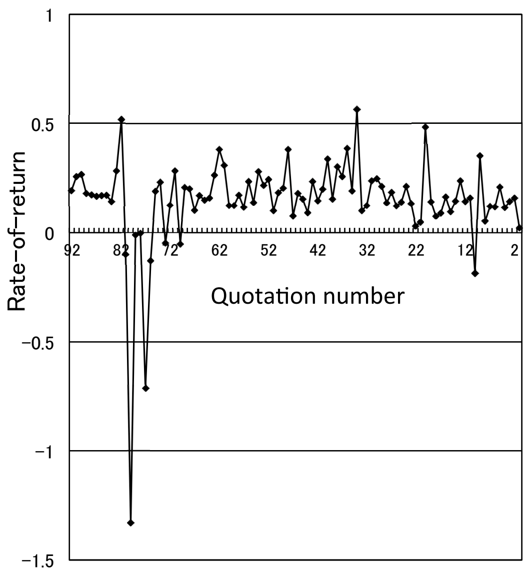

FIGURE 6. Production rate of return of control panel by cost accounting $h_{max} \cong 0.5$

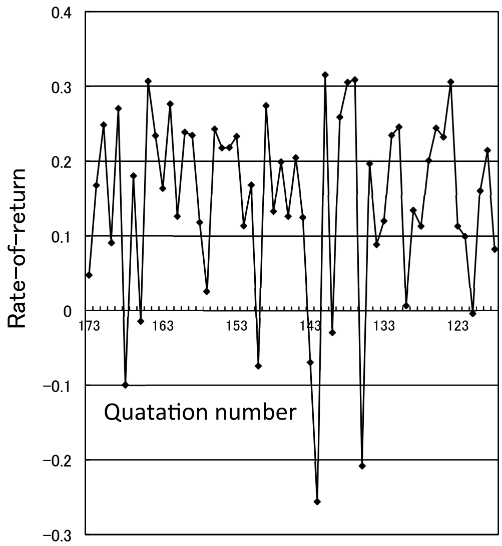

FIGURE 7. Production rate of return of control panel by cost accounting $h_{max} \cong 0.32$

8. **Conclusions.** Using the real data on the rate of return from September 2014 to September 2016, we were able to derive the potential function for the GL potential energy. Moreover, by specifying these parameters, we could obtain the entropy of the three states. With regard to the GL potential energy, the rate of return is influenced by logistical delays, lead times, and breakdown of electronic components. Therefore, we analyzed the parameters of the potential function using the GL free energy. In future research, we plan to apply the potential theory to quality thresholds and quality fluctuations.

REFERENCES

[1] M.Yasuda: Deterministic and simulated annealing approach to fuzzy c-means clustering; *International Journal of Innovative Computing, Information and Control*, Vo.5, No.12(B), pp.4981-pp.4991, December, 2009

[2] Christopher G. Langton: Computation at the edge of chaos; *Physica* D, 42, 1990

[3] K.Shirai, Y.Amano and S.Omatu: Consideration of Phase Transition Mechanisms during Production in Manufacturing Processes; *International Journal of Innovative Computing, Information and Control*, Vol.9, No.9, pp.3611-3626, September, 2013

[4] K.Shirai and Y.Amano: Analysis of fluctuations in production processes using Burgers equation; *International Journal of Innovative Computing, Information and Control*, Vol.12, No.5, pp.1615-1628, October, 2016

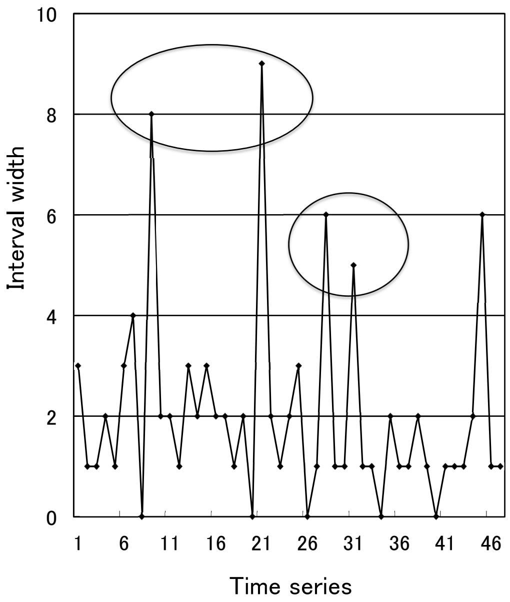

FIGURE 8. Changing interval width with respect to ordering time series from 2014 Sep. to 2014 Dec. ($\{S_f\}$, $average = 2.04$ and $\sigma = 3.7$: before improvement)

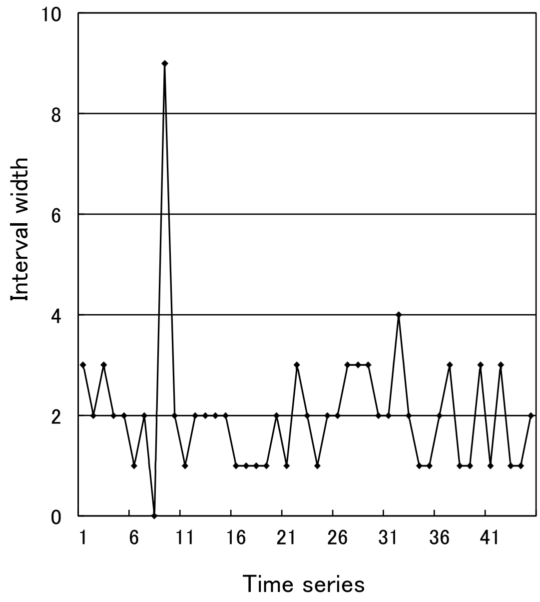

FIGURE 9. Changing interval width with respect to ordering time series from 2014 Sep. to 2014 Dec. ($\{S_i\}$, $average = 2.02$ and $\sigma = 1.8$: after improvement)

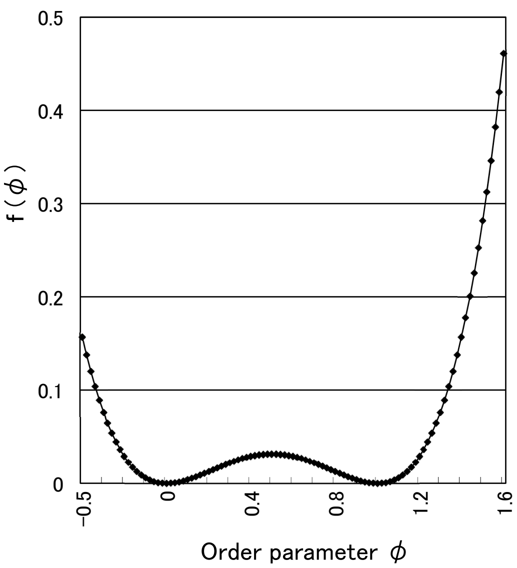

FIGURE 10. Potential function by the phase field method ($a = 0.1$, $b = 0$, $c = 0$)

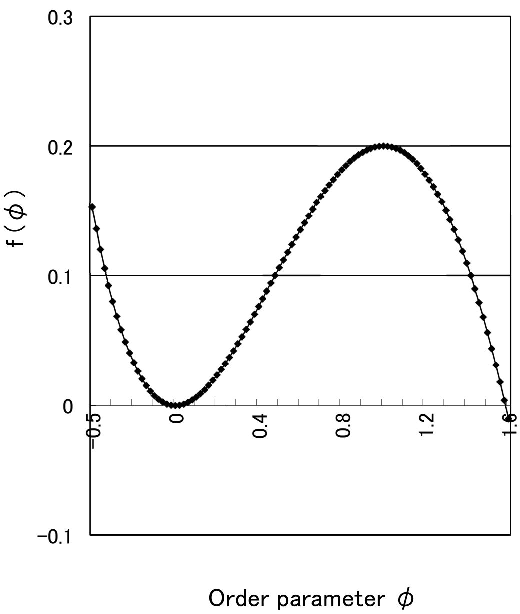

FIGURE 11. Potential function by the phase field method ($a = 0.1$, $b = 0.2$, $c = 0$)

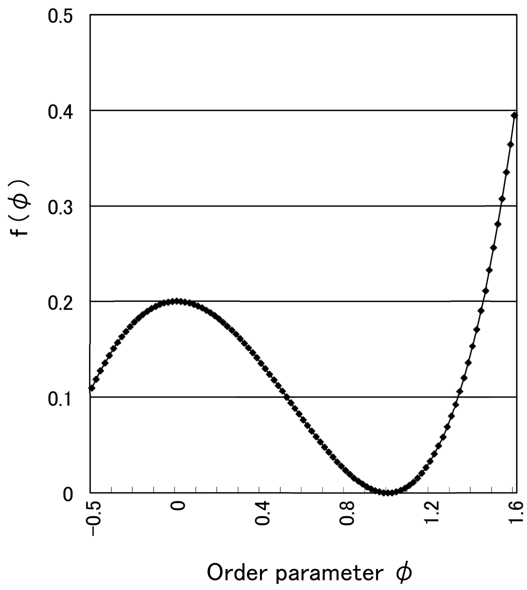

FIGURE 12. Potential function by the phase field method ($a = 0.1$, $b = 0$, $c = 0.2$)

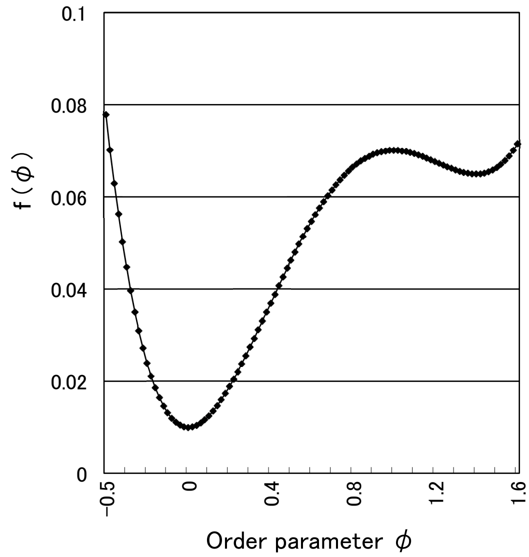

FIGURE 13. Potential function by the phase field method ($a = 0.1$, $b = 0.07$, $c = 0.01$): $\{S_f\}$

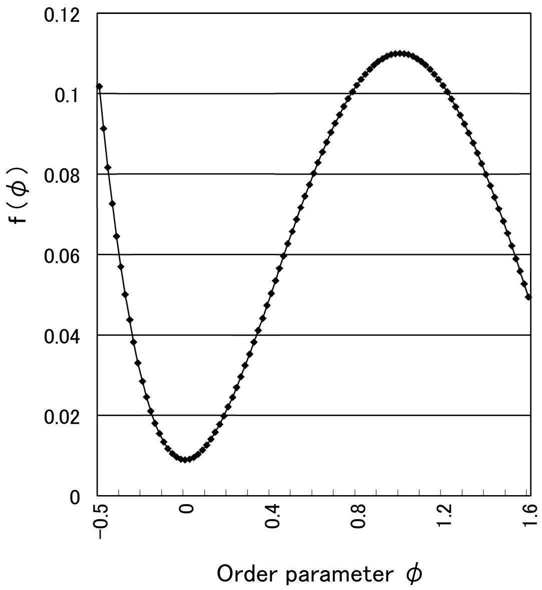

FIGURE 14. Potential function by the phase field method ($a = 0.1$, $b = 0.01$, $c = 0.009$): $\{S_{Local}\}$

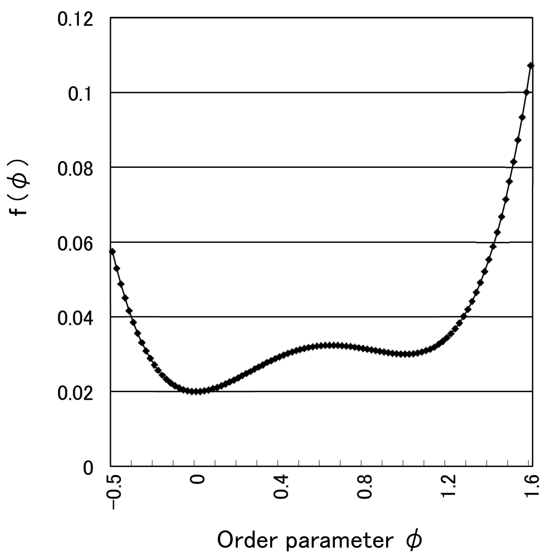

FIGURE 15. Potential function by the phase field method ($a = 0.1$, $b = 0.03$, $c = 0.02$): $\{S_i\}$

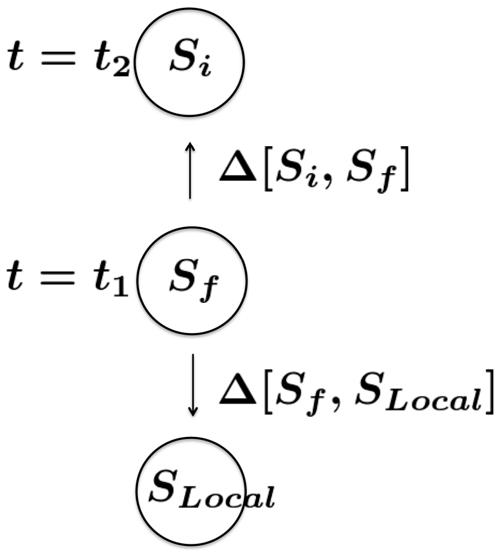

FIGURE 16. Probabilistic representation of process time series

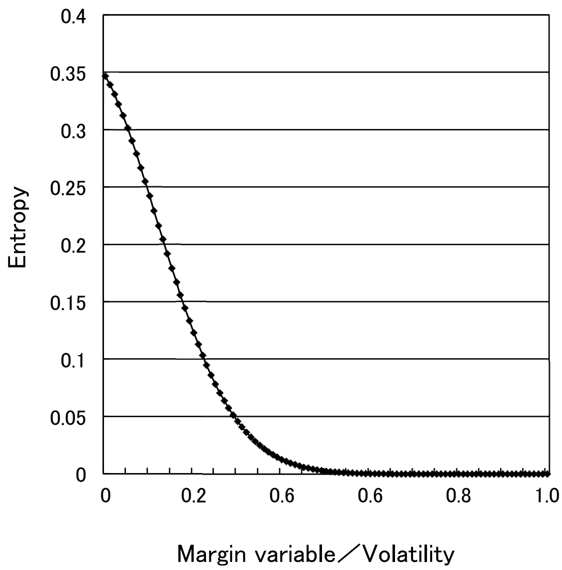

FIGURE 17. Entropy in consideration of standardized volatility $\{S_i\}$

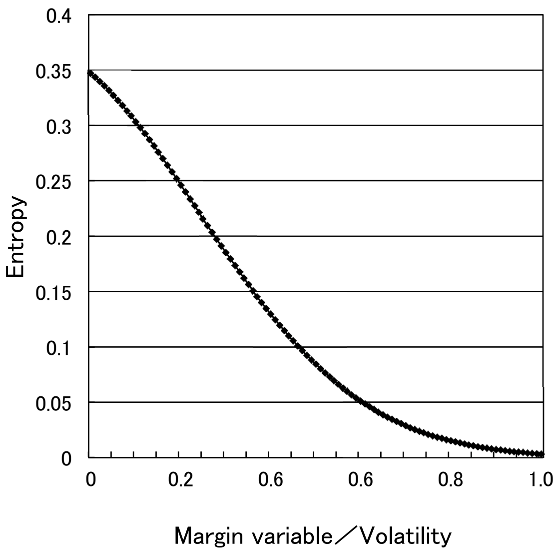

FIGURE 18. Entropy in consideration of standardized volatility $\{S_f\}$

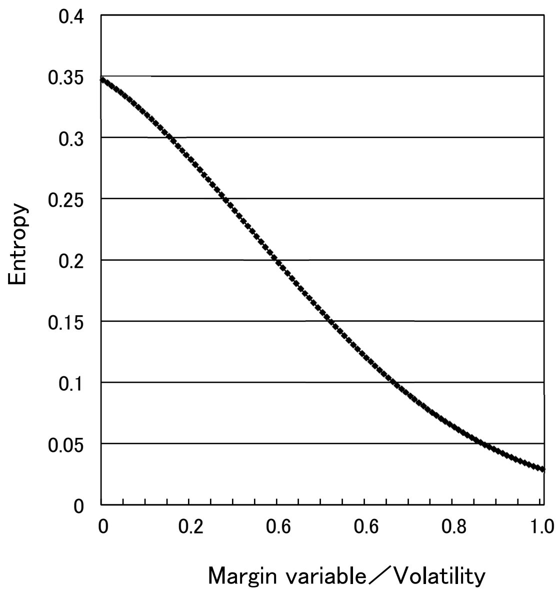

FIGURE 19. Entropy in consideration of standardized volatility $\{S_{Local}\}$

[5] K.Shirai and Y.Amano: Application of an Autonomous Distributed System to the Production Process; *International Journal of Innovative Computing, Information and Control*, Vol.10, No.4, pp.1247-1265, August, 2014
[6] K.Shirai, Y.Amano, and S.Omatu: Improving throughput by considering the production process; *International Journal of Innovative Computing, Information and Control*, Vol.7, No.2, pp.4917-4930, December, 2013
[7] T.Tanabe and M.Ishikawa: Influence of the random disturbances on the chemotactic bacterial colony formations; *International Journal of Innovative Computing, Information and Control*, Vo.4, No.1, pp.25-34, January, 2012
[8] Fuyuto Takase: Multi-mode vibration in the group of transmitters coupled lattice; *KURENAI: Kyoto University Research Information Repository*, Vol.413, pp10-29, 1981
[9] M.Kuramitsu and Y.Nishikawa: A mathematical analysis of self-organization in electrical circuits； *The society of Instrument and Control Engineers*, Vol. 29, No.10, pp.899-904, 1990
[10] M.Kuramitsu and H.Takase: Analysis of multi-degree-of-freedom oscillator with average potential; *IEICE Trans.*, Vol.J66-A No.4, pp.336-343, 1983
[11] K.Kitahara: Nonequilibrium statistical mechanics; *Iwanami co.,LTD*, 2000
[12] R.Yamamoto, T.Nakaturu, K.Miyajima and M.Ishikawa: On the mathematical modeling of order-disorder transition by stochastic partial differential equations; *Yamaguchi Univ. Enjineering Faculty reasarch report*, Vol.50, No.1, pp.45-51, 1999
[13] K.Shirai and Y.Amano: Self-similarity of fluctuations for throughput deviations within a production process; *International Journal of Innovative Computing, Information and Control*, Vo.10, No.3, pp.1001-1016, June, 2014
[14] K.Shirai and Y.Amano: Investigation of the relation between production density and lead-time via stochastic analysis; *International Journal of Innovative Computing, Information and Control*, Vo.13, No.4, pp.1117-1133, August, 2017
[15] K.Shirai, Y.Amano and S.Omatu: Nonlinear characteristics of the Rate of Return in the Production Process; *International Journal of Innovative Computing, Information and Control*, Vol.10, No.2, pp.601-616, April, 2014
[16] H.Nishimori：Spin glass theory and information statistical mechanics; *Iwanami co.,LTD*, 1999
[17] T.Imamura: mathematics in Probability field; *Iwanami co.,LTD*, 1976
[18] H.Hidaka and K.Sato, etc.: Teletraffic characteristics of cellular communication for different types of vehicle motion; *IEICE Trans. commun.*, Vol.E84-B, No.3 pp.558-564, 2001

Appendix A. Analysis of the Tes-trun results.

- (Testrun1)： Because the throughput of each process (S1−S6) is asynchronous, the overall process throughput is asynchronous. In Table 3, we list the manufacturing time (min) of each process. In Table 4, we list the volatility in each process performed by the workers. Finally, Table 3 lists the target times. The theoretical throughput is obtained as $3 \times 199 + 2 \times 15 = 627$(min). In addition, the total working time in stage S3 is 199 (min), which causes a bottleneck. In Fig. 20, we plot the measurement data listed in Table 3, which represents the total working time of each worker (K1−K9). In Fig. 21, we plot the data contained in Table 3, which represents the volatility of the working times.
- (Testrun2)： Set to synchronously process the throughput. The target time listed in Table 5 is 500 (min), and the theoretical throughput (not including the synchronization idle time) is 400 (min). Table 6 presents the volatility of each working process (S1−S6) for each worker (K1−K9).
- (Testrun3)： Introducing a preprocess stage. The process throughput is performed synchronously with the reclassification of the process. As shown in Table 7, the theoretical throughput (not including the synchronization idle time) is 400 (min). Table 8 presents the volatility of each working process (S1−S6) for each worker (K1−K9). On the basis of these results, the idle time must be set to 100 (min). Moreover, the theoretical target throughput (T_{s}') can be obtained using the “ Synchronization with

preprocess " method. This goal is as follows:

$$\begin{aligned} T_s &\sim 20 \times 6(\text{First cycle}) + 17 \times 6(\text{Second cycle}) \\ &\quad + 20 \times 6(\text{Third cycle}) + 20(\text{Previous process}) + 8(\text{Idol} - \text{time}) \\ &\sim 370(min) \end{aligned} \tag{27}$$

The full synchronous throughput in one stage (20 min.) is

$$T_s' = 3 \times 120 + 40 = 400(\text{min}) \tag{28}$$

Using the " Synchronization with preprocess " method, the throughput is reduced by approximately 10%. Therefore, we showed that our proposed " Synchronization with preprocess " method is realistic and can be applied in flow production systems. Below, we represent for a description of the " Synchronization with preprocess ".

In Table.7, the working times of the workers K4, K7 show shorter than others. However, the working time shows around target time. Next, we manufactured one piece of equipment in three cycles. To maintain a throughput of six units/day, the production throughput must be as follows:

$$\frac{(60 \times 8 - 28)}{3} \times \frac{1}{6} \simeq 25(min) \tag{29}$$

where the throughput of the preprocess is set to 20 (min). In eqn. (29), the value 28 represents the throughput of the preprocess plus the idle time for synchronization. Similarly, the number of processes is 8 and the total number of processes is 9 (8 plus the preprocess). The value of 60 is obtained as 20 (min) × 3 (cycles).

Table 2. Correspondence between the table labels and the Test−run number

	Table Number	Production process	Working time	Volatility
Test−run1	Table.3	Asynchronous process	627(min)	0.29
Test−run2	Table.5	Synchronous process	500(min)	0.06
Test−run3	Table.7	" Synchronization with preprocess " method	470(min)	0.03

In Table.2, Test−run3 indicates a best value for the throughput in the three types of theoretical working time. Test−run2 is ideal production method. However, because it is difficult for talented worker, Test−run3 is a realistic method.

The results are as follows. Here, the trend coefficient, which is the actual number of pieces of equipment/the target number of equipment, represents a factor that indicates the degree of the number of pieces of manufacturing equipment.
Test−run1: 4.4 (pieces of equipment)/6 (pieces of equipment) = 0.73
Test−run2: 5.5 (pieces of equipment)/6 (pieces of equipment) = 0.92
Test−run3: 5.7 (pieces of equipment)/6 (pieces of equipment) = 0.95
Volatility data represent the average value of each Test−run.

TABLE 3. Total manufacturing time at each stages for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	15	20	20	25	20	20	20
K2	20	22	21	22	21	19	20
K3	10	20	26	25	22	22	26
K4	20	17	15	19	18	16	18
K5	15	15	20	18	16	15	15
K6	15	15	15	15	15	15	15
K7	15	20	20	30	20	21	20
K8	20	29	33	30	29	32	33
K9	15	14	14	15	14	14	14
Total	145	172	184	199	175	174	181
Deviation		27	39	54	30	29	36

TABLE 4. Volatility of Table3

K1	1.67	1.67	3.33	1.67	1.67	1.67
K2	2.33	2	2.33	2	1.33	1.67
K3	1.67	3.67	3.33	2.33	2.33	3.67
K4	0.67	0	1.33	1	0.33	1
K5	0	1.67	1	0.33	0	0
K6	0	0	0	0	0	0
K7	1.67	1.67	5	1.67	2	1.67
K8	4.67	6	5	4.67	5.67	6
K9	0.33	0.33	0	0.33	0.33	0.33

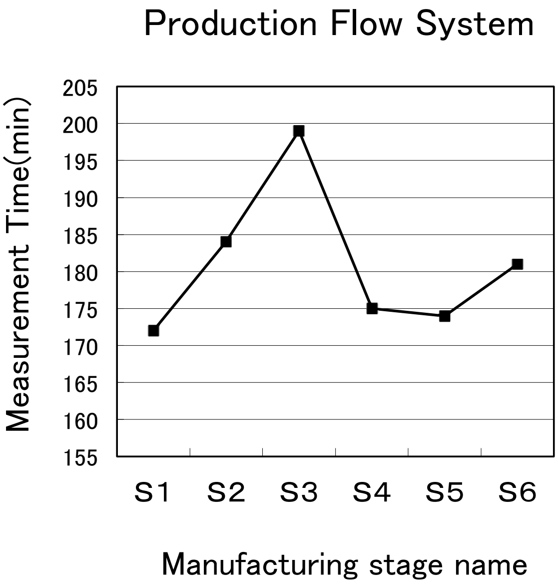

FIGURE 20. Total work time for each stage(S1−S6) in Table 3

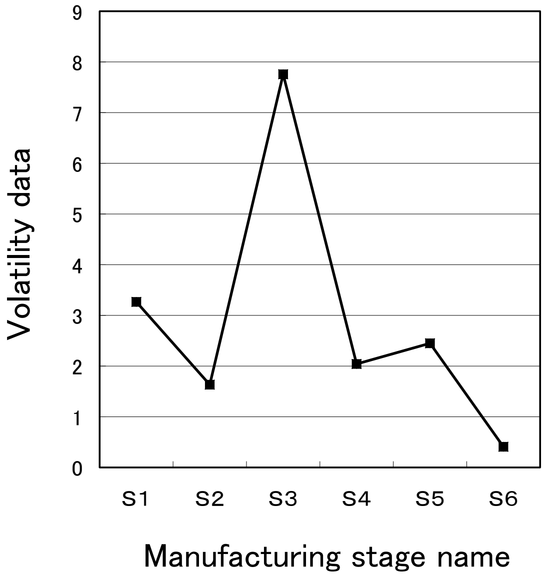

FIGURE 21. Volatility data for each stages(S1−S6) in Table 3

TABLE 5. Total manufacturing time at each stages for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	20	20	24	20	20	20	20
K2	20	20	20	20	20	22	20
K3	20	20	20	20	20	20	20
K4	20	25	25	20	20	20	20
K5	20	20	20	20	20	20	20
K6	20	20	20	20	20	20	20
K7	20	20	20	20	20	20	20
K8	20	27	27	22	23	20	20
K9	20	20	20	20	20	20	20
Total	180	192	196	182	183	182	180
Deviation		12	16	2	3	2	0

TABLE 6. Volatility of Table 5

K1	0	1.33	0	0	0	0
K2	0	0	0	0	0.67	0
K3	0	0	0	0	0	0
K4	1.67	1.67	0	0	0	0
K5	0	0	0	0	0	0
K6	0	0	0	0	0	0
K7	0	0	0	0	0	0
K8	2.33	2.33	0.67	1	0	0
K9	0	0	0	0	0	0

TABLE 7. Total manufacturing time at each stages for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	20	20	20
K2	20	18	18	18	20	20	20
K3	20	21	21	21	20	20	20
K4	20	13	11	11	20	20	20
K5	20	16	16	17	20	20	20
K6	20	18	18	18	20	20	20
K7	20	14	14	13	20	20	20
K8	20	22	22	20	20	20	20
K9	20	25	25	25	20	20	20
Total	180	165	164	161	180	180	180
Deviation		- 15	-16	-19	0	0	0

TABLE 8. Volatility of Table 7

K1	0.67	0.33	0.67	0	0	0
K2	0.67	0.67	0.67	0	0	0
K3	0.33	0.33	0.33	0	0	0
K4	2.3	3	3	0	0	0
K5	1.3	1.3	1	0	0	0
K6	0.67	0.67	0.67	0	0	0
K7	2	2	2.3	0	0	0
K8	0.67	0.67	0	0	0	0
K9	1.67	1.67	1.67	0	0	0